Analysis of the Frequency Locking Region of Coupled Oscillators Applied to 1-D Antenna Arrays

Nidaa Tohmé, Jean-Marie Paillot, David Cordeau, Patrick Coirault

Analysis of the Frequency Locking Region of Coupled Oscillators Applied to 1-D Antenna Arrays

Nidaa Y. Tohmé, Jean-Marie Paillot, David Cordeau, Patrick Coirault

University of Poitiers, LAII
40 avenue du Recteur Pineau, 86022 Poitiers cedex, France
nyaacoub, jmpaillot, dcordeau @iutang.univ-poitiers.fr

***Abstract*— During the past decade, coupled oscillators have shown their efficiency as simple methods for phase control in microwave antenna arrays, and hence as alternatives to conventional electronic beam steering methods. In this paper, a new writing of the nonlinear equations proposed by R. York to describe the oscillators' locked states is presented. This has allowed the elaboration of a CAD tool which provides, in a considerably short simulation time, the frequency locking region of the coupled oscillators. This region is plotted versus the oscillators' tunings referred to the resonant frequency of the coupling circuit. A prototype circuit consisting of a five oscillators array is currently under test to validate the theory.**

***Index Terms*— Beam steering, coupling circuits, design automation, nonlinear oscillators, phased arrays, synchronization.**

I. Introduction

Arrays of coupled oscillators are receiving increasing interest in both military and commercial applications. They are used to achieve high-power RF sources through coherent power combining. Another application is the beam steering of antenna arrays. The radiation pattern of a phased antenna array is steered in a particular direction by establishing a constant phase progression throughout the oscillators chain. For a linear array (Fig. 1), a phase shift $\Delta\phi$ between adjacent elements results in steering the beam to an angle θ off broadside, given by:

$$\theta = \arcsin\left(\frac{\lambda_0}{2\pi d}\Delta\phi\right) \tag{1}$$

where d is the distance separating two antennas and λ_0 is the free-space wavelength [1]. The required inter-element phase shift can be obtained by detuning the free-running frequencies of the outermost oscillators in the array [2]. The resulting phase shift is independent of the number of oscillators in the array [3], [4], [5], [6].

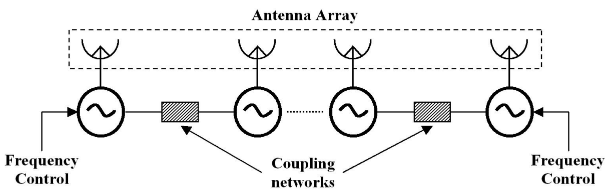

Fig. 1. Block Diagram of an array of N-coupled oscillators.

In his previous works, R. York made use of simple Van der Pol oscillators to model coupled microwave oscillators. Based on a generalization of Kurokawa's method [7], he was able to derive the equations for the amplitude and phase dynamics of N oscillators coupled through many types of circuits [8].

In this paper, mathematical manipulations were applied to the nonlinear equations describing the locked states of the coupled oscillators proposed in [8]. A reduced system of equations was obtained, thus allowing the elaboration of a CAD tool that permits to acquire, in a considerably short simulation time, the frequency locking region of the coupled oscillators, in terms of the amplitudes of their output signals and the phase shift between them.

In subsection II-A, the dynamics of two Van Der Pol oscillators coupled through a resonant network will be overviewed. Then, the reduced system of equations will be introduced in subsection II-B. In section III, the developed CAD tool as well as its reliability test using Agilent's ADS software will be described. Section V is dedicated to illustrate the application of the proposed tool to beam steering. In the last section, preliminary experimental results are presented.

II. Theoretical Analysis

A. Overview of the dynamics of two Van Der Pol oscillators coupled through a resonant network

The theory of coupled microwave oscillators is the subject of increasing research activity. Simple Van der Pol oscillators, coupled through a resonant network that produces a constant magnitude and phase delay between the oscillators, provided a satisfactory model for a lot of applications [3], [8].

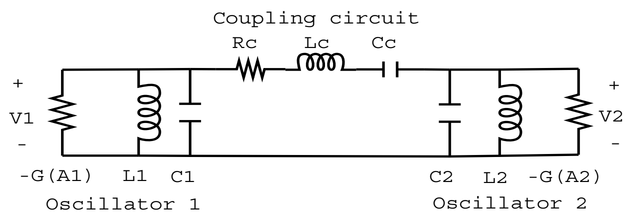

Fig. 2. Two parallel resonant circuits coupled through a series RLC network.

Fig. 2 represents two oscillators coupled through a series resonant circuit. These oscillators are considered identical, except for their free-running frequencies. Thus, this work aims

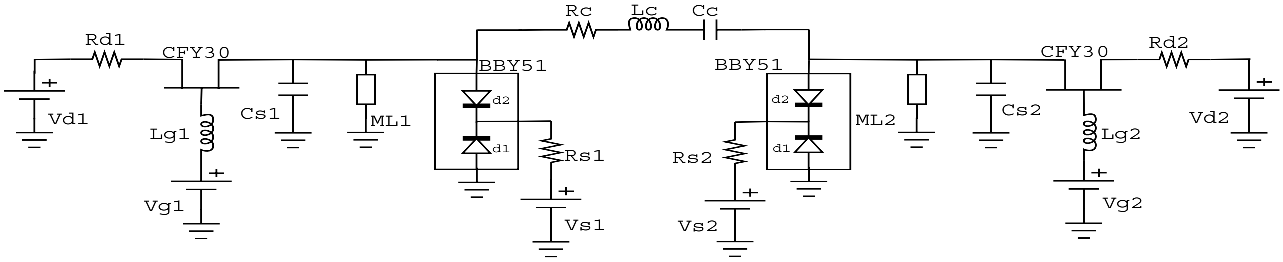

Fig. 3. Two transistor-based ($GaAsFETCFY30$) oscillator circuits, with microstrip lines and varactor diodes ($BBY51$) as resonators, coupled through a series RLC network.

to determine values for these frequencies and for that of the coupling circuit that result in frequency locking.

The ability of these oscillators to lock to a common frequency is affected by the following parameters:

- $\lambda_0 = \frac{1}{G_0 R_c}$: coupling constant, with G_0 being the first-order term of Van Der Pol nonlinear conductance
- $\omega_a = \frac{G_0}{C}$: bandwidth of the oscillators
- $\omega_{ac} = \frac{R_c}{L_c}$: bandwidth of the unloaded coupling circuit
- ω_{01}, ω_{02} : free-running frequencies or tunings of oscillators 1 and 2, respectively
- ω_{0c} : resonant frequency of the coupling circuit

In the following, the free-running frequencies of the oscillators, ω_{01} and ω_{02}, and the synchronization frequency of the system, ω, are referred to the resonant frequency of the coupling circuit, ω_{0c}, using the substitutions below

$$\Delta\omega_{01} = \omega_{01} - \omega_{0c} \quad \Delta\omega_{02} = \omega_{02} - \omega_{0c} \quad \Delta\omega_c = \omega - \omega_{0c}.$$

Starting from the admittance transfer functions binding the coupling current to the oscillators voltages, and relying on Kurokawa's substitution [7], J. Lynch and R. York described the oscillators dynamic equations, as well as those for the amplitude and phase of the coupling current. Then, by setting the derivatives equal to zero, the algebraic equations describing the oscillators frequency locked states were obtained [8].

In the following subsection, mathematical manipulations are applied to these equations leading to a new writing of the oscillators' locked states.

B. New writing of the equations describing the locked states of coupled oscillators

Because of the trigonometric and strongly non linear aspect of the equations in [8], thus making their numeric resolution a hard issue, a simpler system of three equations with three unknowns was established. It is presented as follows:

$$A_1^2 \left(1 - X_1 - A_1^2\right)^2 \left(1 + \frac{Y_1}{1 - X_1 - A_1^2}\right) = X_2^2 A_2^2 \qquad (2)$$

$$A_2^2 \left(1 - X_1 - A_2^2\right)^2 \left(1 + \frac{Y_2}{1 - X_1 - A_2^2}\right) = X_2^2 A_1^2 \qquad (3)$$

$$\left(Y_1 + Y_2\right) \frac{\Delta\omega_c^2}{\omega_{ac}^2} + 2\left(Y_1 Y_2 - 1\right) \frac{\Delta\omega_c}{\omega_{ac}} = \left(Y_1 + Y_2\right) \qquad (4)$$

where

$$\begin{aligned} X_1 &= \frac{\lambda_0}{1 + \frac{\Delta\omega_c^2}{\omega_{ac}^2}} \\ X_2 &= \frac{\lambda_0}{\sqrt{1 + \frac{\Delta\omega_c^2}{\omega_{ac}^2}}} \\ Y_1 &= \frac{\Delta\omega_{01} - \left(1 - X_1 \frac{\omega_a}{\omega_{ac}}\right) \Delta\omega_c}{\omega_a \left(1 - X_1 - A_1^2\right)} \\ Y_2 &= \frac{\Delta\omega_{02} - \left(1 - X_1 \frac{\omega_a}{\omega_{ac}}\right) \Delta\omega_c}{\omega_a \left(1 - X_1 - A_2^2\right)} \end{aligned}$$

The solutions of the above equations are then used to evaluate the corresponding inter-element phase shift expressed as follows:

$$\Delta\phi = \arctan\left[\frac{\frac{\Delta\omega_{01}}{\omega_a} + \frac{\Delta\omega_c}{\omega_{ac}}\left(1 - A_1^2 - \frac{\omega_{ac}}{\omega_a}\right)}{1 - \lambda_0 - A_1^2 - \frac{\Delta\omega_c^2}{\omega_a \omega_{ac}}\left(1 + \frac{\Delta\omega_{01}}{\Delta\omega_c}\right)}\right] \qquad (5)$$

III. CAD Tool

The system of equations presented above was implemented on Matlab and a CAD tool was developed allowing to obtain, in a considerably short simulation time, the cartography of the locked states of the coupled oscillators.

In this paper, the oscillator's structure used in simulation is depicted in Fig. 3. For each one of these oscillators, the transistor used is the GaAs FET CFY30, and the resonator circuit is composed of a microstrip short-circuited stub in parallel with a varactor diode, allowing to easily control the free-running frequency of the oscillator, according to the required phase shift.

In order to represent the oscillator circuit of Fig. 3 by a negative resistance in parallel with an LC resonator as shown in Fig. 2, Agilent's ADS software was used. From ADS simulation results for one oscillator at the required synchronization frequency, it is then possible to extract the parallel LC circuit that models the resonator, as well as the parameters a_1 and a_3 of the Van Der Pol equation capable of reproducing the behavior of the oscillator's active part.

Considering now two oscillators coupled through a series RLC network as in Fig. 3. Using ADS, this system can be

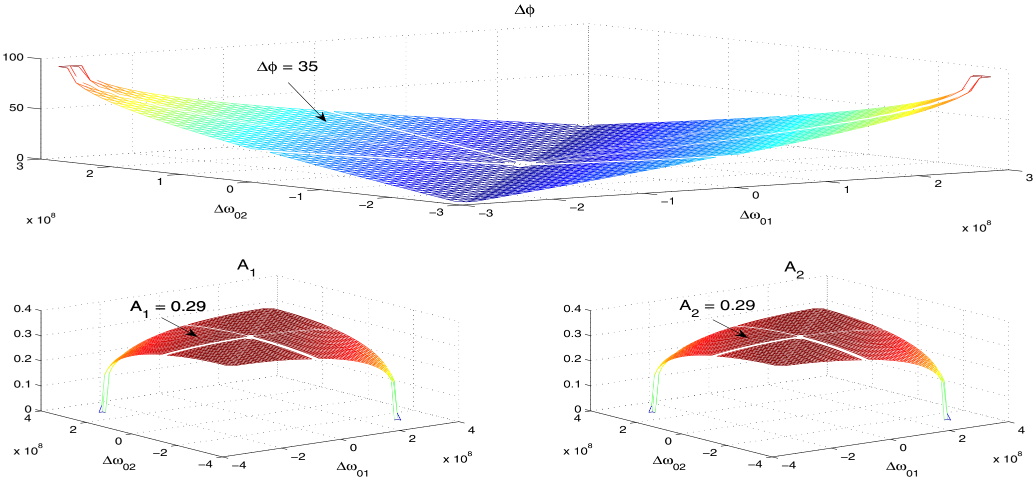

Fig. 4. Cartography of the oscillators' locked states provided by the CAD tool.

reduced into two Van Der Pol oscillators with LC resonators as shown in Fig. 2. Then, driven by the corresponding bandwidths of these oscillators and their coupling circuit, as well as the coupling strength, the developed CAD tool provides the user with the range of frequencies over which these two oscillators can lock. For instance, for a synchronization frequency of $975\,MHz$, with a ratio of bandwidths $\omega_{ac}/\omega_a = 13$ and a coupling constant $\lambda_0 = 1.1$, the cartography of the oscillators' locked states provided by the CAD tool is presented in Fig. 4. It illustrates, respectively, the variations of the phase shift, $\Delta\phi$, and the oscillators amplitudes, A_1 and A_2, in function of $\Delta\omega_{01}$ and $\Delta\omega_{02}$.

The validation of our results starts by comparing them to the results obtained with Agilent's ADS software. With the latter, only a transient analysis of one point at a time of the synchronisation region allows to verify the synchronization results obtained with the CAD tool. The amplitudes of the oscillators' output signals and the phase shift between them is provided for each combination of $(\Delta\omega_{01}, \Delta\omega_{02})$. For instance, the point marked with an arrow in the three subplots of Fig. 4 corresponds to a free-running frequency $f_{01} = 950\,MHz$ for oscillator 1 and $f_{02} = 1\,GHz$ for oscillator 2. The corresponding resonant frequency of the RLC coupling circuit is $f_{0c} = 972\,MHz$. To these points correspond a phase shift of $35°$ and an amplitude of $288\,mV$ at the output of each of the coupled oscillators.

This same coupled system simulated with ADS has lead to two sinusoidal waves at $973.4\,MHz$ and $38°$ out of phase, with an amplitude of $312\,mV$ at the output of each oscillator (Fig. 5).

Then, and under the same simulation conditions, five oscillators were coupled together through the same RLC circuit. The resulted waveforms are shown in Fig. 6. The average phase shift between adjacent oscillators was found to be equal to $37°$.

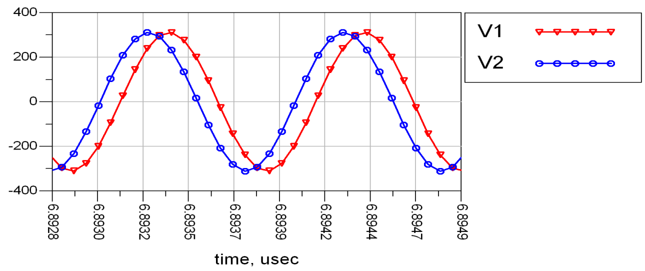

Fig. 5. Waveforms present at the output of each oscillator in the case of 2 oscillators array.

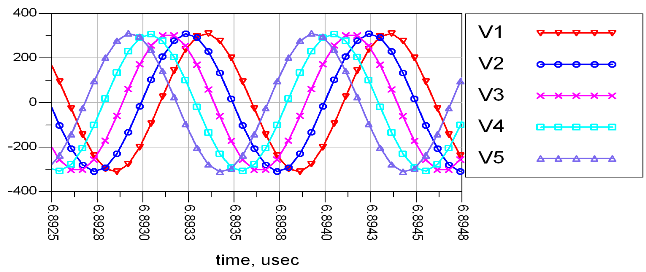

Fig. 6. Waveforms present at the output of each oscillator in the case of 5 oscillators array.

Thus, these results show that it will be possible to adjust, with a high accuracy, the free-running frequencies required to achieve the desired phase shift in an N-element array.

The main advantage of this CAD tool is that, in an extremely short simulation time, one can obtain all achievable phase shifts and the free-running frequencies, f_{01} and f_{02}, required to generate them.

IV. Application to Beam Steering

As mentioned in the introduction, steering the beam of a linear array to an angle θ off broadside requires a constant phase progression $\Delta\phi$ between adjacent elements as given in (1).

Let us consider a simple array of two antennas separated by a distance $d = \lambda/2$. The circuit presented in the previous section (Fig. 3) is used to control the radiation pattern of this array. Thus, and under the same simulation conditions used in the previous section, the cartography of the synchronization zone of the coupled oscillators is generated using the developed CAD tool (Fig. 4). This cartography provides all characteristics, i.e. the amplitudes of the oscillators' output signals and the phase shift between them, which describe the behaviour of coupled oscillators versus $(\Delta\omega_{01}, \Delta\omega_{02})$.

From this cartography one can extract the values of $(\Delta\omega_{01}, \Delta\omega_{02})$ which corresponds to the 35° phase shift found in section III. These were found to be $\Delta\omega_{01} = -1.38 \times 10^8 rds^{-1}$ and $\Delta\omega_{02} = 1.76 \times 10^8 rds^{-1}$, and consequently, the free-running frequencies to impose are $f_{01} = 950\,MHz$ for oscillator 1 and $f_{02} = 1\,GHz$ for oscillator 2, assuming a resonant frequency of $972\,MHz$ for the coupling circuit. Hence, using (1), the steer angle achievable with these conditions is $\theta = 11.2°$.

V. Experimental results

In order to experimentally validate the developed tool, a prototype circuit consisting of 5 coupled oscillators was realized. A photo of the PCB is shown in Fig. 7. In each oscillator circuit, a transistor ATF35143 was used instead of the CFY30, since the latter was not available. The varactor diode used in each resonator is the BBY51. Thus, the free running frequency of each oscillator was varied by modifying the command voltage of its varactor diode.

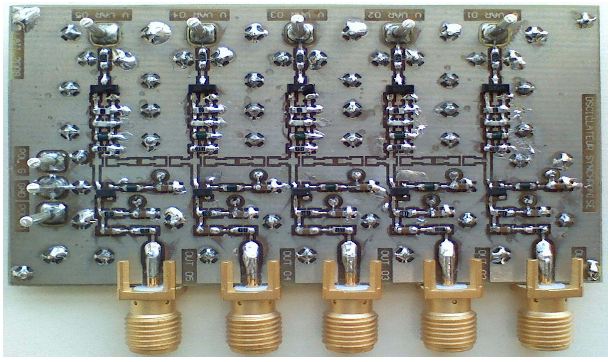

Fig. 7. Prototype circuit with 5 coupled oscillators.

The first measurements were performed with two coupled oscillators only. The tuning characteristics of both VCO's are shown in Fig. 8. The resulting frequency variation band is equal to 140 MHz approximately.

In these measurement conditions, the phase shift obtained between the two coupled oscillators was comprised between -38.12 ° and +39.14 ° .

Currently, the efforts are focalized on the implementation of a method of characterization to test the phases of the five oscillators.

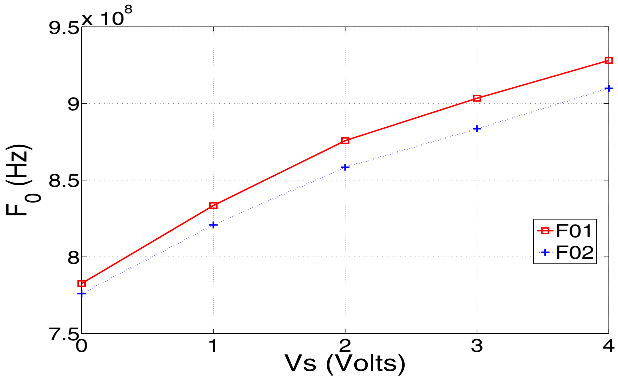

Fig. 8. Tuning characteristics of both oscillators.

VI. Conclusions

In this paper, mathematical manipulations were applied to the system of nonlinear equations proposed in [8] describing the locked states of two Van der Pol oscillators coupled through a resonant network. A simpler system of three equations with three unknowns was obtained allowing then to implement a CAD tool that permits to extract the locking region of the coupled oscillators, in an extremely short simulation time, and then to study the effect of the frequencies and bandwidths of the oscillators and coupling circuit, as well as the coupling strength, on the ability of the oscillators to lock.

The reliability of the developed CAD tool was verified using Agilent's ADS software, where two transistor-based oscillators coupled through a series RLC circuit were simulated. The obtained results were in accordance with those generated using the CAD tool.

Preliminary measurements performed on a prototype circuit has proven the promising efficiency of the developed tool for a lot of applications, and namely for beam steering.

References

[1] W. Stutzman and G. Thiele, *Antenna Theory and Design*. New York: John Wiley and Sons, 1981.

[2] P. Liao and R. York, "A new phase-shifterless beam-scanning technique using arrays of coupled oscillators," *IEEE Trans. Microwave Theory Tech.*, vol. 41, pp. 1810–1815, Oct. 1993.

[3] R. York, "Nonlinear analysis of phase relationships in quasi-optical oscillator arrays," *IEEE Trans. Microwave Theory Tech.*, vol. 41, pp. 1799–1809, Oct. 1993.

[4] P. Liao and R. York, "Beam scanning with coupled vcos," *IEEE Proc. Antennas Propagation Soc. Int. Symp.*, vol. 2, pp. 836–839, June 1994.

[5] R. York and T. Itoh, "Injection and phase-locking techniques for beam control," *IEEE Trans. Microwave Theory Tech.*, vol. 46, pp. 1920–1929, Nov. 1998.

[6] P. Maccarini, J. Buckwalter, and R. York, "Coupled phase-locked loop arrays for beam steering," *IEEE MTT-S Int. Microwave Symp. Dig.*, vol. 3, pp. 1689–1692, June 2003.

[7] K. Kurokawa, "Injection locking of solid state microwave oscillators," in *Proc. IEEE*, vol. 61, no. 10, Oct. 1973, pp. 1386–1409.

[8] J. Lynch and R. York, "Synchronization of oscillators coupled through narrow-band networks," *IEEE Trans. Microwave Theory Tech.*, vol. 49, pp. 237–249, Feb. 2001.